USE U SUBSTITUTION FOR X

1) $x^4 - 10x^2 + 9 = 0$ factor and take half of 1st exponent

$u = x^2$ $u^2 - 10u + 9 = 0$

$(u-9)(u-1)=0$

$u = 9, 1$

$x^2 = 9$ $x^2 = 1$

$x = \sqrt{9}$ $x = \sqrt{1}$

the solution set is $\boxed{-3, 3, -1, 1}$

2) $2x^4 - x^2 - 1 = 0$

$u = x^2$ $2u^2 - u - 1 = 0$ *use slide and divide*

$u^2 - u - 2 = 0$

$(u-2)(u+1)=0$ *divide both by 2*

$u = 1, -\frac{1}{2}$ *can't take square root of a negative number*

$x^2 = 1$ $\cancel{x^2 = -\frac{1}{4}}$

$x = \boxed{\pm 1}$

Another 2) $21x^4 - 4x^2 - 1 = 0$

$u = x^2$ $21u^2 - 4u - 1 = 0$ *use slide and divide*

$u^2 - 4u - 21 = 0$

$(u-7)(u+3)=0$ *divide both by 21*

$u = \frac{7}{21} = \frac{1}{3}$ $u = -\frac{3}{21} = -\frac{1}{7}$ *can't take square root*

$x^2 = \frac{1}{3}$ $\cancel{x^2 = -\frac{3}{7}}$ *of a negative number*

$x = \pm\sqrt{\frac{1}{3}} = \boxed{\frac{\sqrt{3}}{3}}$

3) $x^6 - 26x^3 - 27 = 0$ $u=x^3$ $u^2-26u-27=0$ take half of 1^{st} exponent

$(u-27)(u+1)=0$ set both equal to zero

$u = -1,27$

$x^3 = -1$ $x^3 = 27$

the solution set is $\boxed{-1,3}$

4) $(x+2)^2 + 9(x+2) + 14 = 0$ u is always the middle term

$u= \boxed{x+2}$ $\boxed{u^2+9u+14=0}$

$(u+7)(u+2)=0$

$u = -7,-2$ PUT X BACK IN FOR U

$x+2 = -7$ and $x + 2 = -2$

the solution set is $\boxed{-9, -4}$

5) $(x^2-2x)^2 - 27(x^2-2x) + 72 = 0$ u is always the middle term

Let u $\boxed{x^2-2x}$ then the equation in us is $\boxed{u^2-27u+72=0}$

$(u-3)(u-24)=0$

$u = 3, 24$

$x^2 - 2x = 3$ and $x^2 - 2x = 24$

$x^2 - 2x - 3 - 0$ and $x^2 - 2x - 24 = 0$

$(x-3)(x+1)=0$ and $(x-6)(x+4)=0$

the solution set is $\boxed{3, -1, 6, -4}$

6) $(2x+4)^2 + 4(2x+4) + 4 = 0$ u is always the middle term

$u=2x+4$ the given equation with correct substitution is $u^2+4u+4=0$

*if nothing is in front of the middle term just use u

$(u+2)(u+2)=0$

$u = -2$

$2x+4 = -2$

the solution set is $\boxed{-3}$

7) $(x+8)^2 + 10(x+8) + 9 = 0$ u is always the middle term

u=x+8 the given equation with correct substitution is $u^2+10u+9=0$

*if nothing is in front of the middle term just use u $(u+9)(u+1)=0$

$u = -9,-1$

$x+8 = -9$ and $x+8 = -1$

the solution set is -17, -9

8) $2(s+7)^2 - 13(s+7) = 7$ u= s+7 $2u^2-13u-7=0$ *slide and divide*

$u^2-13u-14=0$

$(u-14)(u+1)=0$ divide by 2

$u = 7, \frac{-1}{2}$

$s+7 = 7$ and $s+7 = \frac{-1}{2}$

the solution set is $\boxed{0, -\frac{15}{2}}$

9) $y = (x+2)^2 - 11(x+2) + 28$ u= x+2 $u^2-11u+28=0$

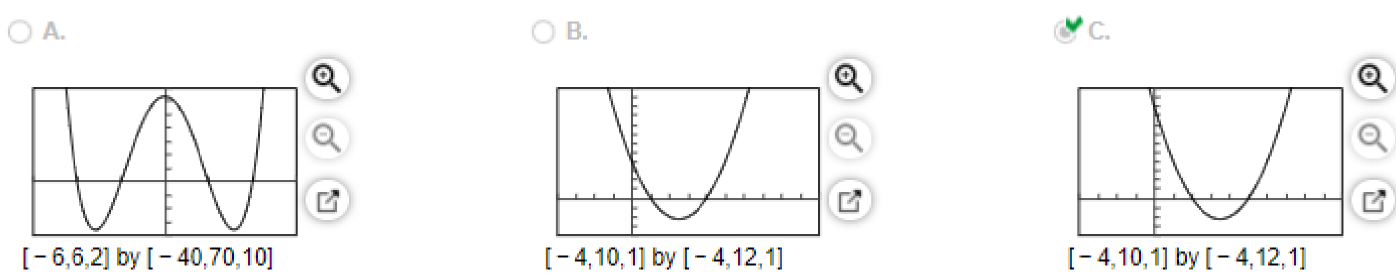
$(u-7)(u-4)=0$

$u = 7,4$

$x+2 = 7$ and $x+2 = 4$

the x-intercepts are $\boxed{5, 2}$

Choose the correct graph below. look at the x-intercepts to find graph



10) 6(x+2) +5a(x+2) Factor out the (x+2)

$\boxed{(x+2)(6+5a)}$

11) $5x(8x^6+9)-4(8x^6+9)$ Factor out the $(8x^6+9)$

$\boxed{(8x^6+9)(5x-4)}$

12) $3x^2+4xy+6x+8y$ Factor by grouping: take GCF from each highlighted part

$x(3x+4y)+2(3x+4y)$ then factor out the $(3x+4y)$

$\boxed{(3x+4y)(x+2)}$

13) $3x^2+3xy-7x-7y$ Factor GCF from each highlighted part

$3x(x+y)-7(x+y)$ then factor out the $(x+y)$

$\boxed{(x+y)(3x-7)}$

14) Watch the section lecture video and answer the question listed below. Note: The counter in the lower right corner of the screen displays the Example number.

As shown in Examples 1-3, what two things have to be true in order to use the zero factor property?

[Click here to watch the video.]

Select all that apply.

- [ ] A. Both sides of the equation must be a polynomial.
- [ ] B. One side of the equation must be a polynomial not in factored form.
- [x] C. One side of the equation must be a factored polynomial.
- [x] D. One side of the equation must be zero.
- [ ] E. One side of the equation must be one.

15) Find real solutions by factoring $x^3-36x=0$

Real solutions means set each $x(x^2-36)=0$ factor out x first

$x(x+6)(x-6)=0$ factor difference of two squares

$x=0,\ x+6=0,\ x-6=0$

$\boxed{0,-6,6}$

16) Find real solutions by factoring $5x^3 = 2x^2$ move everything to the left

$5x^3 - 2x^2=0$ Factor out x^2

$x^2(5x-2)=0$ set each part =0

$\boxed{0,\frac{2}{5}}$

17) Find real solutions by factoring $x^3 - 13x^2 + 42x = 0$

$x(x^2-13x+42)=0$ factor out x first then trinomial

$x(x-7)(x-6)=0$ set each part =0

$\boxed{0,6,7}$

18) Find real solutions by factoring $x^3 + 8x^2 - 64x - 512 = 0$

Factor GCF from each highlighted part $x^2(x+8)- 64(x+8)=0$

$(x^2-64)(x+8)=0$

$(x+8)(x-8)(x+8)=0$

do not duplicate answers the solution set is $\boxed{-8,8}$

19) Find real solutions by factoring $x^3 - 8x^2 - 9x + 72 = 0$

Factor GCF from each highlighted part $x^2(x-8) - 9(x-8)=0$

Factor out the (x-8) $(x^2- 9)(x-8)=0$

Factor difference of two squares $(x+3)(x-3)(x-8)=0$

do not duplicate answers the solution set is $\boxed{-3,3,8}$

20) Find real solutions by factoring $2x^3 + 16 = x^2 + 32x$ move everything to the left

Factor GCF from each highlighted part $2x^3 - x^2 - 32x + 16=0$

Factor out the (6x-5) $x^2(2x - 1)- 16(2x - 1)=0$

$(x^2-16)(2x-1)=0$

Factor difference of two squares $(x+4)(x-4)(2x-1)=0$

$x+4-0$ $x-4=0$ $2x-1=0$

the solution set $\boxed{-4,4,\frac{1}{2}}$

QUIZ EXAMPLES

1) Zeros are 0 and 2 of multiplicity 2, f(3) = 18

Put into factored form: $y = x(x-2)^2$

plug in 3 for x and 18 for y and solve for a

$18 = a(3)(3 - 2)^2$

$18 = 3a \quad a = 6$

then write equation with a = 6

$y = 6x(x-2)^2$

Expand the equation: $y = 6x(x^2 - 4x + 4)$

$y = 6x^3 - 24x^2 + 24x$

2) Zeros are -3, -1, 4 f(-2) = -18

Put into factored form: $y = (x+3)(x+1)(x-4)$

plug in -2 for x and -18 for y and solve for a

$-18 = a(-2+3)(-2+1)(-2-4)$

$-18 = 6a \quad a = -3$

then write equation with a = 6

$y = -3(x+3)(x+1)(x-4)$

Expand the equation: $y = (-3x - 9)(x^2 - 3x - 4)$

$y = -3x^3 + 9x^2 + 12x - 9x^2 + 27x + 26$

$y = -3x^3 + 39x + 26$

EXTRA PROBLEMS

a) $x - 12x\sqrt{x} = 0$ move one term to the right

$(x = 12x\sqrt{x})^2$ square both sides

$x^2 = 144x^3$ move back to left to factor

$x^2 - 144x^3 = 0$

$x^2\,(1-144x) = 0$ set each part equal to zero

$x^2 = 0$ and $1 - 144x = 0$

$x = 0, \frac{1}{144}$

b) $x + \sqrt{x} = 72$ $u = \sqrt{x}$ $u^2+u-72=0$

$(u-8)(u+9)=0$ $u = -9, 8$

$\sqrt{x} = -9$ $\sqrt{x} = 8$

no solution $x = 64$

c) $4x^{\frac{1}{2}} - 9x^{\frac{1}{4}} + 3 = 0$ $u = x^{\frac{1}{4}}$ $4u^2-9u+3=0$ *Use quadratic*

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\frac{9 \pm \sqrt{81-4\cdot4\cdot3}}{8}$ $u = \frac{9 \pm \sqrt{33}}{8}$

The opposite of ¼ root is raised to the 4^{th} power, put answer in exactly like this

$x = (\frac{9+\sqrt{33}}{8}^4), (\frac{9-\sqrt{33}}{8}^4)$

d) $(\sqrt[4]{7x^2-6} = x\ )^4$ raise both sides to the 4^{th} power

$7x^2-6 = x^4$ move everything to the right side0

$x^4 - 7x^2 - 6 = 0$ FACTOR, half of 1^{st} exponent

$(x^2-6)(x^2-1)=0$ set both sides equal to zero

$x^2-6=0$ and $x^2-1=0$ take square root of both

$x = \sqrt{6}, 1$ ** if $x = \sqrt{8}$ must reduce radical $\sqrt{2\cdot4}$ → $x = 2\sqrt{2}$*

e) $x^2 + 8x + 3\sqrt{x^2+8x} = 18$ u=$\sqrt{x^2+8x}$ $u^2+3u-18=0$

$(u-3)(u+6)=0$ u = -6,3

$\sqrt{x^2+8x}$ = -6 $\sqrt{x^2+8x}$ = 3

can't have negative $x^2+8x=9$ move 9 to left to factor

$x^2+8x-9=0$ FACTOR

(x+9)(x-1)=0 set both sides equal to zero

x = -9,1

f) $x^{-2} - 6x^{-1} + 8 = 0$ $u=x^{-1}$ $u^2-6u+8=0$

(u-2)(u-4)=0 u = 2,4

$x^{-1}=2$ $x^{-1}=4$ negative exponent makes answer a fraction

x = ½, ¼

g) $5x^{2/3} - 34x^{1/3} - 7 = 0$ $u=x^{1/3}$ $5u^2-34u-7=0$ USE SLIDE AND DIVIDE

$u^2-34u-35=0$

(u-35)(u+1)=0 divide by 5

u = 7, -1/5

$x^{1/3} = 7$ $x^{1/3} = -1/5$

opposite of 1/3 exponent is cube so cube both answers

x = 343, - $\frac{1}{125}$

h) $\left(\frac{v}{v+1}\right)^2 + \frac{3v}{v+1} = 18$ u=$\frac{v}{v+1}$ the 3 is the coefficient in front of u

$u^2+3u-18=0$

(u+6)(u-3)=0 u = -6,3

$\frac{v}{v+1}$ = -6 $\frac{v}{v+1}$ = 3

-6v-6=v 3v+3=v

-6 = -7v 3 = -2v

x = $-\frac{6}{7}$ and $-\frac{3}{2}$

j) Find real solutions by factoring $x^3 - 2x^2 + 49x - 98 = 0$

Factor GCF from each highlighted part $x^2(x-2)+49(x-2)=0$

$(x^2+49)(x-2)=0$

$\cancel{x^2+49}= 0$ and $x-2 = 0$

square can never = 0 $x^2= -49$ *no solution* **$x = 2$**

k) Find real solutions by factoring $6x^3 + 36x = 5x^2 + 30$

move everything to the left

Factor GCF from each highlighted part $6x^3 - 5x^2 - 36x - 30=0$

Factor out the (6x-5) $x^2(6x - 5)- 6(6x - 5)=0$

$(x^2-6)(6x-5)=0$

$x^2-6= 0$ and $6x-5 = 0$ $x=-\sqrt{6}, \sqrt{6}, \frac{5}{6}$